

This assignment covers the whole module.

You should answer **ALL FOUR** questions. One of the requirements for passing the module is that you must obtain a score of at least 30 out of 100 for this assignment.

Question 1 – 25 marks

On 1 September 2011, a fictitious lake in Norway has a population of 500 000 adult crayfish, increasing at a rate of approximately 7.4% per year without human intervention. The lake can sustain a population of 700 000 adult crayfish. It is proposed that each summer 40 000 adult crayfish should be harvested. Let A_n denote the size of the population on 1 September n years after 2011.

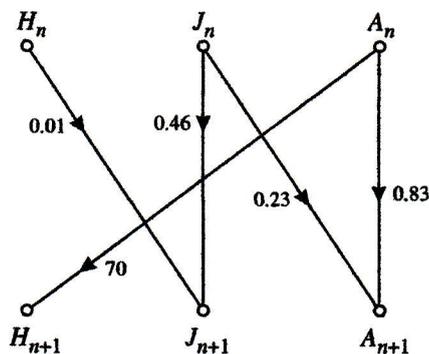
- (a) It is suggested that the effect on the population size can be modelled by the recurrence system

$$A_0 = 500\,000, \quad A_{n+1} = 1.074 A_n - 40\,000 \quad (n = 0, 1, 2, \dots).$$

- (i) Briefly explain how this recurrence system is obtained from the information given at the beginning of the question. [2]
- (ii) Use the recurrence system to find the population size predicted by the model for 1 September 2013 (that is, two years after 2011). [2]
- (iii) Calculate the equilibrium population size according to the model. [2]
- (iv) Find a closed form for the sequence A_n given by the model, and hence find the population size predicted for 1 September 2016. What is the behaviour of the sequence A_n for large n ? [4]
- (v) Use logarithms to find the year when the adult crayfish population is predicted to be extinct by 1 September, according to the model, if the level of harvesting remains unchanged. [4]
- (b) Adult crayfish (4 years or older) form a subpopulation of the crayfish in the lake, which include hatchlings (under 1 year) and juveniles (1–3 years). Let H_n and J_n denote the sizes of the subpopulations of hatchlings and juveniles, respectively, on 1 September n years after 2011. The following table presents the data for the three subpopulations.

	Hatchlings	Juveniles	Adults
Subpopulation on 1 September 2011	30 000 000	400 000	500 000
Birth rate	0	0	70
Death rate	0.99	0.31	0.17
Proposed catch	0	0	40 000

The network diagram below illustrates the interdependency of the three subpopulations in the absence of harvesting. It assumes that in each year all of the surviving hatchlings become juveniles, and that a third of surviving juveniles become adults.



- (i) Explain clearly, giving both calculations and the reasoning leading to the calculations, how each of the numbers 0.01, 0.46 and 0.23 in the network diagram are derived from the description of the model and the data in the table. [6]
- (ii) The recurrence system below is obtained by converting the network diagram into a matrix model, taking into account the initial data for 2011 and the effect of the proposed annual catch. It is suggested as an alternative model to that in part (a) for the effect of the proposed annual catch on the number of adult crayfish.

$$\begin{pmatrix} H_0 \\ J_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} 30\,000\,000 \\ 400\,000 \\ 500\,000 \end{pmatrix}$$

$$\begin{pmatrix} H_{n+1} \\ J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 70 \\ 0.01 & 0.46 & 0 \\ 0 & 0.23 & 0.83 \end{pmatrix} \begin{pmatrix} H_n \\ J_n \\ A_n \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 40\,000 \end{pmatrix}$$

Calculate the adult crayfish subpopulation size predicted by the model for 1 September 2012. [3]

- (iii) According to Mathcad, the model in part (b) predicts that the subpopulation size of adult crayfish in 2031 will be 863 666, which is greater than the maximum sustainable population of the lake.

Give a reason why the model in part (b) does not give an accurate prediction for the adult crayfish subpopulation size in 2031. [2]

Question 2 – 25 marks

This question requires the use of Mathcad throughout, and no marks will be awarded to answers obtained by other means. For each of parts (a)–(c) you should provide an appropriate printout, though a printout on one page may cover your answers to several parts. Annotate your printouts with Mathcad text or handwriting, or reference them from a separate sheet, in order to explain clearly what you have done and what your conclusions are.

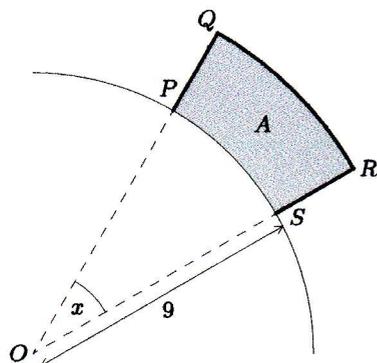
You may find it useful in all parts of this question to refer to A Guide to Mathcad.

A flexible wire $PQRS$, of total length 12 metres, is bent into a three-edged planar shape, and its ends P, S are placed against a disc of radius 9 metres with centre O , as shown in the diagram below. (The arc PS is not part of the wire.) The end-segments PQ and RS of the wire lie along straight lines through O , while the arc QR forms part of a circle with centre O and subtends an angle x (in radians) at O .

This question concerns the area A enclosed between the wire and the edge of the disc, which is shown shaded below. This area can be expressed by $A = f(x)$, where

$$f(x) = \frac{9x(4 - 3x)(16 + 3x)}{2(2 + x)^2} \quad (0 \leq x \leq \frac{4}{3}).$$

(You are NOT asked to show this.)



In part (a), you may either use the Mathcad graph plotter file (121A3-04) or plot the graph in a new worksheet of your own.

- (a) (i) Plot the graph of the function $f(x)$. Your graph should cover the interval $[0, 1.33]$ in the x -direction and $[0, 20]$ in the y -direction. [4]
- (ii) By using the 'Trace' facility (and also, if you wish, the 'Zoom' facility), estimate to two decimal places the coordinates of the point on this graph at which $y = f(x)$ takes its maximum value. [2]
- (iii) On the same graph, plot the line $y = 8$. Using the 'Trace' facility, estimate to two decimal places both solutions of the equation $f(x) = 8$. (These solutions give the values of x for which the shaded area is 8 m^2 .) [3]

- (b) Each of the following recurrence relations has the property that if a sequence generated by the recurrence relation converges to a limit in the interval $[0, \frac{4}{3}]$, then that limit must be a solution of the equation $f(x) = 8$. (You are NOT asked to show this.)

$$\mathbf{A}: x_{n+1} = \frac{16(2 + x_n)^2}{9(4 - 3x_n)(16 + 3x_n)} \quad (n = 0, 1, 2, \dots)$$

$$\mathbf{B}: x_{n+1} = \frac{8}{32 - 3\sqrt{76 - 9x_n}} \quad (n = 0, 1, 2, \dots)$$

- (i) For each of these recurrence relations, generate the sequence with starting value $x_0 = 1$, and tabulate your results to six decimal places. Which sequence converges more rapidly? (That is, which sequence gives an estimate with specified accuracy for its limit with the smaller value of n ?) [5]
- (ii) Use your tabulated results to write down the solutions of the equation $f(x) = 8$ to six decimal places. [3]
- (c) This part of the question concerns finding the maximum value of the function $f(x)$, as estimated in part (a)(ii), and hence the maximum possible value of the shaded area A .

You may need to put $x := x$ in your worksheet before answering part (c)(i).

- (i) Use symbolic differentiation and the 'simplify' keyword to find an expression for the derivative $f'(x)$. [3]
- (ii) The maximum value of $f(x)$ occurs where $f'(x) = 0$. Use a solve block to solve the equation $f'(x) = 0$ for x , giving your answer to six decimal places. [3]
- (iii) Calculate the corresponding maximum possible value of the area A , giving your answer to four decimal places. [2]

Question 3 – 25 marks

As for other questions, remember to show your working explicitly throughout your answer to this question.

- (a) (i) Use the Composite Rule to differentiate the function

$$f(x) = e^{\frac{1}{2}x + \cos x}. \quad [5]$$

- (ii) Use the Product Rule and your answer to part (a)(i) to show that the function

$$g(x) = (1 + 2 \sin x) e^{\frac{1}{2}x + \cos x} \quad (0 \leq x \leq 2\pi)$$

has derivative

$$g'(x) = \frac{1}{2} (4 \cos^2 x + 4 \cos x - 3) e^{\frac{1}{2}x + \cos x}. \quad [4]$$

- (iii) Find any stationary points of the function $g(x)$ defined in part (a)(ii), and use the First Derivative Test to classify each stationary point as a local maximum or a local minimum of $g(x)$.

(Note that the domain of the function g is the interval $[0, 2\pi]$, and recall that $|\cos x| \leq 1$ for all values of x .)

[5]

- (iv) Using your answers to parts (a)(ii) and (a)(iii), find the area below the graph of

$$y = (1 - 2 \cos x)(3 + 2 \cos x) e^{\frac{1}{2}x + \cos x} \quad (0 \leq x \leq 2\pi)$$

and above the x -axis. Give your answer to five significant figures.

[4]

- (b) (i) Using your answer to part (a)(i), find the general solution of the differential equation

$$\frac{dy}{dx} = (2 \sin x - 1) e^{\frac{1}{2}(x+y) + \cos x},$$

giving the solution in implicit form.

[4]

- (ii) Find the particular solution of the differential equation in part (b)(i) for which $y = -2$ when $x = 0$, and then give this particular solution in explicit form.

[3]

Question 4 – 25 marks

This question requires the use of OUStats.

- (a) The file AGES.OUS contains data on the ages in months of 113 students enrolled on an elementary statistics course. This part of the question concerns these data. [Lindgren, B. W. and Berry, D. A. (1981) *Elementary Statistics*, Macmillan, page 67.]
- (i) Use OUStats to find the age of the youngest student enrolled on the course. What facility of OUStats did you use to find this value? [2]
 - (ii) Obtain a printout of a *histogram* of the data using starting value 210 months and an interval width 50 months. State, with a brief explanation, a reason why a normal distribution might not be a suitable model for the variation of these data. [4]
 - (iii) Obtain a printout of a *frequency diagram* of the data using starting value 200.5 months and interval width 20 months. [2]
 - (iv) Making use of the frequency diagram obtained in part (a)(iii), write down the most popular age interval for the enrolled students. How many enrolled students are in this age interval? Find the number of enrolled students who are 25 years old exactly or less. [4]
- (b) The file ALLOYS.OUS contains data related to thirteen specimens of cupro-nickel alloys, which were tested for corrosion in salt water for 60 days. 'Iron' contains the iron content of each specimen (the units are not given in the source) and 'Loss' contains the corresponding weight loss in mg per square decimetre per day. The data are paired. [Draper, N. R. and Smith, W. L. (1966) *Applied Regression Analysis*, John Wiley, page 37.]
- (i) Obtain a printout of a scatterplot of 'Iron' and 'Loss', using 'Iron' as the explanatory variable (which must be on the x -axis). Add the least squares fit line to the plot. [3]
 - (ii) Describe any pattern that you observe on the plot. [2]
 - (iii) Obtain the equation of the least squares fit line of 'Loss' on 'Iron'. If you use labels other than 'Iron' and 'Loss', specify which variable represents 'Iron' and which represents 'Loss'. [2]
 - (iv) Using the model from part (b)(iii), estimate the mean weight loss due to corrosion for a cupro-nickel alloy with iron content of 0.1. [2]
 - (v) The first observation has iron content of 0.01 and weight loss due to corrosion of 127.6 mg per square decimetre per day. Using the model from part (b)(iii), find the residual for this observation. [2]
 - (vi) Briefly explain why it would be inappropriate to use the model from part (b)(iii) to estimate the mean weight loss due to corrosion for a cupro-nickel alloy with iron content of 2.3. [2]
-

This CMA contains 28 questions.

Questions 1 to 7 are on Block A.

Question 1

Choose the TWO options that are false statements.

Options

- A $(2\sqrt{2})^6 = 8^3$
- B $\frac{(3a)^2 \times b^3 \times c^2}{(2a)^4 \times b \times c^3} = \frac{9b^2}{16a^2c} \quad (a \neq 0, b \neq 0, c \neq 0)$
- C $\sqrt{7} = 2.6457$, correct to four decimal places.
- D $\sqrt{24} = 2\sqrt{6}$
- E $\sqrt{5}$ cannot be expressed as a recurring decimal.
- F $(\frac{1}{4})^{-5/2} = -32$
-

Question 2

A sequence is defined by

$$u_n = 3 \times (-0.8)^n + 5 \quad (n = 0, 1, 2, \dots).$$

Choose the option that describes the long-term behaviour of the sequence.

Options

- A u_n becomes arbitrarily small (that is, $u_n \rightarrow 0$ as $n \rightarrow \infty$).
- B u_n becomes arbitrarily large and positive (that is, $u_n \rightarrow \infty$ as $n \rightarrow \infty$).
- C u_n approaches 5 (that is, $u_n \rightarrow 5$ as $n \rightarrow \infty$).
- D u_n is unbounded and alternates in sign.
- E u_n becomes arbitrarily large and negative (that is, $u_n \rightarrow -\infty$ as $n \rightarrow \infty$).
- F u_n approaches 3 (that is, $u_n \rightarrow 3$ as $n \rightarrow \infty$).
-

Question 3

Choose the option that gives the equation of the line that passes through the point $(-2, 4)$ and is perpendicular to the line $3x - y = 5$.

Options

- A $3x - y = -10$ B $x + 3y = 10$ C $3x - y = 5$
- D $3x + y = -2$ E $x - 3y = 5$ F $3x + y = 10$
- G $x + 3y = -5$ H $x - 3y = -10$
-

Question 4

The method of completing the square can be used to write the expression $5x^2 - 6x + 7$ in the form $a(x + b)^2 + c$, where a , b and c are constants. Choose the option that gives the value of c .

Options

- A $-\frac{44}{5}$ B -2 C $\frac{44}{25}$ D $\frac{26}{5}$ E $\frac{166}{25}$ F $\frac{866}{25}$
-

Question 5

Choose the option that gives the domain of the function

$$f(x) = \frac{1}{\sqrt{(4-x)(x+1)}}$$

Options

- A $[-1, 4)$ B $(-\infty, 4]$ C $(-4, 1)$
 D $(-1, 4)$ E \mathbb{R} F $[-1, \infty)$
-

Question 6

Choose the option that is a **false** statement.

Options

- A $\arcsin(\tan(-\frac{1}{3}\pi)) = -\frac{1}{3}\pi$
 B $\cos(\arccos(-\frac{1}{2}\sqrt{3})) = -\frac{1}{2}\sqrt{3}$
 C $\arcsin(\sin(-1)) = -1$
 D $\arccos(\sin(-\frac{1}{4}\pi)) = \frac{3}{4}\pi$
 E $\arcsin(1) = 2 \arctan(1)$
 F $\cos(\arcsin(-1)) = 0$
-

Question 7

Choose the **TWO** options that are **true** statements.

Options

- A $2^{3 \log_2 7} = 343$
 B If $2^x = 70$, then $x = \frac{\ln 70}{\ln 2} = 6.12$, correct to 3 significant figures
 C $2 \ln x + \ln y - \ln 5 = \ln\left(\frac{2xy}{5}\right)$ ($x, y > 0$)
 D $\log_{10} \sqrt{100} = 10$
 E $e^{\ln 3} = 3e$
 F $\ln\left(\frac{2x^2 - 8}{e^{3x}}\right) = \ln 2 + \ln(x + 2) - 3x + \ln(x - 2)$ ($x > 2$)
-

Questions 8 to 14 are on Block B.

Question 8

Choose the option that gives the value of $\sum_{i=39}^{81} (43 + 5i)$.

Options

- A 4429 B 6534 C 14 344 D 14 706
E 14 749 F 18 454 G 22 064 H 27 649
-

Question 9

Choose the TWO options that give the term P_1 and the long-term behaviour for the sequence generated by the recurrence system

$$P_0 = 2450, \quad P_{n+1} = P_n + 0.84P_n \left(1 - \frac{P_n}{8575}\right) \quad (n = 0, 1, 2, \dots).$$

Options

- A 1470 B 3920 C 4200 D 4900
E The sequence converges to 8575 with values always below 8575.
F The sequence converges to 8575 with values eventually alternating above and below 8575.
G The sequence tends to a 2-cycle.
H The sequence shows chaotic behaviour.
-

Question 10

Choose the option that gives the sum of the infinite series

$$\left(\frac{4}{9}\right)^3 - \left(\frac{4}{9}\right)^4 + \left(\frac{4}{9}\right)^5 - \left(\frac{4}{9}\right)^6 + \dots$$

Options

- A $\frac{4}{729}$ B $\frac{16}{1161}$ C $\frac{16}{387}$ D $\frac{64}{1053}$
E $\frac{16}{117}$ F $\frac{64}{405}$ G $\frac{64}{351}$ H $\frac{4}{13}$
-

Question 11

The variation in size of a population is modelled by the recurrence relation

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} 0.9217 & 0.0213 \\ 0.0663 & 0.9872 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix} - \begin{pmatrix} 0.007 \\ 0.043 \end{pmatrix},$$

where J_n and A_n are the sizes, in millions, of the subpopulations of juveniles and adults, respectively, n years after 1 January 2011. On that date there were 2.43 million juveniles and 11.78 million adults. Choose the option that gives the total population size, in millions, to two decimal places, predicted by the model for 1 January 2012.

Options

- | | | | |
|----------------|----------------|----------------|----------------|
| A 13.82 | B 14.23 | C 14.28 | D 14.33 |
| E 14.65 | F 14.70 | G 14.75 | H 15.12 |
-

Question 12

Three forces (and no others) act upon an object, which remains at rest. The forces are represented by the vectors \mathbf{P} , \mathbf{Q} and \mathbf{R} , where

$$\mathbf{P} = 5\mathbf{i} + 4\mathbf{j} \quad \text{and} \quad \mathbf{Q} = -3\mathbf{i} + 6\mathbf{j}.$$

Choose the **TWO** options that give the magnitude and direction of the vector \mathbf{R} , to one decimal place.

Options

- | | |
|-----------------------------------|-----------------------------------|
| A Magnitude 3.7 | B Magnitude 10.2 |
| C Direction -168.7° | D Direction -101.3° |
| E Direction -78.7° | F Direction -11.3° |
| G Direction 11.3° | H Direction 78.7° |
-

Question 13

The vector \mathbf{a} has magnitude 7.4 and direction -112° , and the vector \mathbf{b} has magnitude 2.1 and direction -26° . Choose the option that gives the component form of the vector $2\mathbf{a} - 3\mathbf{b}$, to one decimal place.

Options

- | | | |
|---|--|--|
| A $-11.2\mathbf{i} - 11.0\mathbf{j}$ | B $-8.4\mathbf{i} - 4.1\mathbf{j}$ | C $-7.3\mathbf{i} - 12.8\mathbf{j}$ |
| D $0.2\mathbf{i} - 11.0\mathbf{j}$ | E $3.6\mathbf{i} + 8.2\mathbf{j}$ | F $7.3\mathbf{i} + 4.1\mathbf{j}$ |
| G $10.4\mathbf{i} - 12.8\mathbf{j}$ | H $11.2\mathbf{i} + 11.0\mathbf{j}$ | |
-

Question 14

With respect to the usual convention for labelling a triangle (shown in Figure 3.1(a) on page 30 of Chapter B3), the triangle ABC has side lengths $a = 9$ and $b = 5$, and angle $A = 42^\circ$. Choose the option that gives the angle B of the triangle, to one decimal place.

Options

- A 4.3° B 17.7° C 21.8°
D 158.2° E 162.3° F 175.7°
G It could be either 4.3° or 175.7° .
H It could be either 21.8° or 158.2° .
-

Questions 15 to 21 are on Block C.

Questions 15 and 16

The function f is given by

$$f(x) = \ln(e^x + e^{-x} + 2).$$

By applying in turn the Composite and Quotient Rules, choose the option that gives an expression for each of the following.

15 $f'(x)$

16 $f''(x)$

Options for Questions 15 and 16

- A $\frac{1}{e^x - e^{-x}}$ B $\frac{2}{e^x + e^{-x} + 2}$ C $\frac{e^x + e^{-x}}{e^x + e^{-x} + 2}$
D $\frac{e^x - e^{-x}}{e^x + e^{-x} + 2}$ E $\frac{2(e^{2x} + e^{-2x})}{(e^x + e^{-x} + 2)^2}$ F $\frac{2(e^x + e^{-x})}{(e^x + e^{-x} + 2)^2}$
G $\frac{e^x + e^{-x}}{(e^x - e^{-x})^2}$ H $\frac{e^x - e^{-x}}{e^{2x} + e^{-2x}}$
-

Question 17

By first applying the Quotient Rule, choose the option that gives (correct to two decimal places) the overall maximum value of the function

$$f(x) = \frac{15x^2}{7 + x^3}$$

on the interval $[1, 7]$.

Options

- A 1.88 B 2.10 C 4.15 D 4.88
E 5.09 F 5.32 G 6.73 H 7.51
-

Question 18

By using *either* equation (2.3) *or* equation (2.4) in Chapter C2, choose the option that gives an expression for the indefinite integral

$$\int \frac{\sin(3x)}{\sqrt{2 - \cos(3x)}} dx \quad (x > 0).$$

In each option, c is an arbitrary constant.

Options

- | | |
|---|---|
| A $\frac{2}{3}\sqrt{2 - \cos(3x)} + c$ | B $\frac{1}{3}\cos(3x)\sqrt{2 - \cos(3x)} + c$ |
| C $\frac{\sqrt{2 - \cos(3x)}}{\sin(3x)} + c$ | D $-\frac{3\sin^2(3x)}{2 - \cos(3x)} + c$ |
| E $\frac{1}{6}\ln(2 - \cos(3x)) + c$ | F $3\ln(\sqrt{2 - \cos(3x)}) + c$ |
| G $\ln\left(\frac{\sin^2(3x)}{\sqrt{2 - \cos(3x)}}\right) + c$ | H $\ln(\sin(3x)) + \frac{1}{2}\ln(2 - \cos(3x)) + c$ |
-

Question 19

Choose the option that gives (correct to two decimal places) the area of the region which is bounded by the x -axis and by the graph of

$$y = \frac{2(8 - x)(x - 1)}{x^2}.$$

Options

- | | | | |
|---------------|---------------|---------------|---------------|
| A 1.05 | B 1.99 | C 2.60 | D 3.14 |
| E 4.51 | F 5.36 | G 7.82 | H 9.43 |
-

Question 20

Choose the option that gives the solution of the initial-value problem

$$\frac{dy}{dx} = 6y^{2/3}\sin^2\left(\frac{1}{2}x\right) \quad (y > 0), \quad y = 8 \text{ when } x = 0.$$

Options

- | | |
|--|--|
| A $y = 4(1 + \cos^2(\frac{1}{2}x))$ | B $y = (1 + \cos x)^3$ |
| C $y = 8(1 + \sin^2(\frac{1}{2}x))$ | D $y = (4 + x - \sin x)^{3/2}$ |
| E $y = 8 + (x - \sin x)^3$ | F $y = (2 + x - \sin x)^3$ |
| G $y = \left(\frac{8}{1 + \cos x}\right)^{3/2}$ | H $y = \frac{16}{1 + \cos^2(\frac{1}{2}x)}$ |
-

Question 21

The mass m of a radioactive substance present at time t years can be modelled by the equation $m = m_0e^{-kt}$, where m_0 is the initial mass of the substance and k is a constant. After 57 years, three fifths of the substance has decayed (and therefore two fifths remain). Choose the option that gives (in years, to the nearest year) the half-life of the substance.

Options

- A** 31 **B** 33 **C** 35 **D** 37
E 39 **F** 41 **G** 43 **H** 45
-

Questions 22 to 28 are on Block D.

Question 22

Two dice each have faces numbered 1 to 6. They are rolled together. Choose the option that gives the probability that the difference of the numbers obtained is 1.

Options

- A** $\frac{5}{36}$ **B** $\frac{2}{9}$ **C** $\frac{5}{18}$
D $\frac{5}{12}$ **E** $\frac{1}{2}$ **F** $\frac{17}{18}$
-

Question 23

You will need to use OUStats to answer Question 23.

For a particular insurance policy, the lifetime of the policyholders in years is modelled by a normal distribution with mean 68.8 and standard deviation 5.3. One feature of this policy is a payment to the policyholder on his or her 65th birthday. Choose the option that gives the percentage of policyholders who will receive this payment, according to the model.

Options

- A** 14% **B** 20% **C** 24%
D 76% **E** 80% **F** 86%
-

Questions 24 and 25

The distribution of the weights of the contents of packs of tea bags, labelled as containing 125 g, has mean 126.1 g, and standard deviation 3.2 g.

- 24** Choose the option that is closest to the standard error of the mean weight (in grams) of the contents for samples of 50 packs.

Options for Question 24

- A** 0.06 **B** 0.45 **C** 0.51
D 0.61 **E** 0.79 **F** 1.79

You will need to use OUStats to answer Question 25.

- 25** The mean weight (in grams) of the contents for samples of 30 packs of the same tea bags has standard error 0.58 g. Choose the option that is closest to the probability that the mean weight of the contents for a sample of 30 packs of tea bags is between 124.5 g and 125.5 g.

Options for Question 25

- A** 0.15 **B** 0.20 **C** 0.25
D 0.30 **E** 0.75 **F** 0.85
-

Question 7

A popular pastime in a particular club is flipping beer mats balanced on the edge of a table and trying to catch them before they land back on the table. Ann is successful two times out of five in the long run. Choose the option that is closest to the number of times, on average, that she has to flip a beer mat in order to catch it.

Options

A 0.4 B 0.5 C 1 D 1.2 E 2 F 2.5

Question 8

A game uses a pack of five cards numbered 1, 2, 3, 4, 5. In each round the cards are shuffled, the top card is turned face up, and the number is recorded. This card is then returned to the pack. Choose the option that gives, on average, the number of rounds to two decimal places that would be required to obtain four different numbers.

Options

A 5.42 B 6.42 C 7.42
D 8 E 8.42 F 10.42

Question 9

Assuming that a birth is equally likely to occur on any day of the week, choose the option that is closest to the probability that, in a family of four children, at least two of the children were born on the same day of the week.

Options

A 0.02 B 0.10 C 0.35
D 0.45 E 0.55 F 0.65

Questions 10 to 13 are on Chapter D2.

Question 10

A sample of concentrations of transferrin receptor for six women with laboratory evidence of overt iron-deficiency anaemia yielded the following data, in ppm.

12.1 5.3 7.7 5.4 5.9 13.2

Choose the option that is closest to the sample standard deviation.

Options

A 3.19 B 3.52 C 8.30
D 12.23 E 24.96 F 27.34

Questions 26 and 27

A study examined the durability of two brands of tyres by using a machine with a metallic device that wore down the tyres. The time it took (in hours) for each tyre to blow out was recorded. A summary of the data is given in the table below.

	Sample size	Sample mean	Sample standard deviation
Brand 1	15	3.84	0.59
Brand 2	18	4.20	0.39

- 26** Choose the option that gives the estimated standard error (ESE) for the difference between the two population means, correct to two decimal places.

Options for Question 26

- A** 0.03 **B** 0.13 **C** 0.18
D 0.25 **E** 0.28 **F** 0.51

- 27** The two-sample z -test is to be used to determine whether there is a difference, at the 5% significance level, between the mean blow-out times of the two brands of tyres. Choose the **TWO** options that give the magnitude of the test statistic, to two decimal places, and the conclusion obtained from the test.

Options for Question 27

- A** 0.01 **B** 0.02 **C** 1.10
D 1.20 **E** 2.00 **F** 2.02
G There seems to be a significant difference between the mean blow-out times of Brand 1 and Brand 2 tyres, at the 5% significance level.
H There seems to be no significant difference between the mean blow-out times of Brand 1 and Brand 2 tyres, at the 5% significance level.
-

Questions 11 to 13

You should use OUStats for these questions.

The weights (in kg) of people in a sample selected at random from a particular population are normally distributed with mean 73.15 and standard deviation 6.6.

- 11 Choose the option that is closest to the weight above which approximately 25% of the weights of people from the population will lie.

Options for Question 11

- A 55.20 B 55.87 C 56.63
D 60.24 E 68.70 F 77.60

- 12 Choose the option that is closest to a range of values, symmetric about the mean, within which approximately 95% of the weights of people will lie.

Options for Question 12

- A (60.2, 86.1) B (60.6, 85.7) C (62.3, 84.0)
D (66.9, 88.4) E (62.6, 83.7) F (61.6, 84.7)

- 13 Choose the option that is closest to the percentage of people with weights below 90 kg.

Options for Question 13

- A 1% B 10% C 19%
D 81% E 90% F 99%
-

Questions 14 to 17 are on Chapter D3.

Questions 14 to 16

The distribution of the weights of the contents of boxes of a certain breakfast cereal, labelled as containing 450 g, has mean 452.1 g and standard deviation 6.5 g.

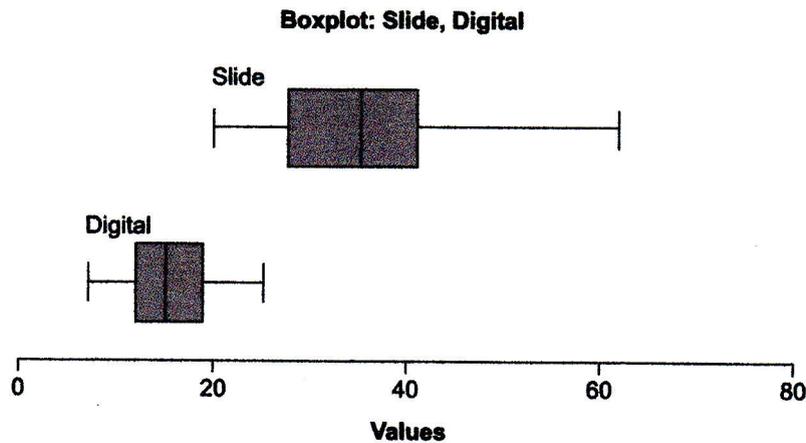
- 14 Choose the option that is closest to the standard error of the mean contents (in grams) for samples of 30 cereal boxes.

Options for Question 14

- A 0.22 B 0.32 C 1.19
D 1.21 E 1.30 F 2.5

Question 28

The comparative effectiveness of conventional and digital image libraries was investigated by comparing the times (in seconds) that thirteen computer-proficient medical professionals took to retrieve the same image from a library of slides and from a computer database with a web interface. Boxplots for the data are shown below, where 'Slide' and 'Digital' refer to the slide library and to the database, respectively.



Choose the option that *must* be a true statement if the boxplots are correct.

Options

- A More data were collected for 'Slide' than for 'Digital'.
 - B The range for 'Slide' is equal to the range for 'Digital'.
 - C The interquartile range is greater for 'Slide' than for 'Digital'.
 - D The mean retrieval time for 'Slide' is about 35 seconds.
 - E The lower quartile is smaller for 'Slide' than for 'Digital'.
 - F The upper quartile is smaller for 'Slide' than for 'Digital'.
-

You should use OUStats for Questions 15 and 16.

The sampling distribution of the mean weight of contents, for samples of 35 cereal boxes, has mean 452.1 g and standard deviation 1.1 g.

- 15 Choose the option that is closest to the probability that the mean weight of the contents of a sample of 35 cereal boxes will be less than 450 g.

Options for Question 15

- A 0.03 B 0.17 C 0.31
D 0.53 E 0.79 F 0.97

- 16 Choose the option that gives a range of values, symmetric about the mean, within which the mean weight of contents of approximately 90% of samples of 35 cereal boxes will lie.

Options for Question 16

- A (449.2, 455.0) B (449.4, 454.8) C (449.9, 454.3)
D (450.3, 453.9) E (451.0, 453.2) F (451.4, 452.8)
-

Question 17

The mean monthly expenditure on petrol per household in Milton Keynes is estimated by selecting a random sample of 36 households. The sample mean is £186.25, and the sample standard deviation is £47.40.

Choose the option that gives an approximate 95% confidence interval for the mean monthly expenditure on petrol per household in Milton Keynes.

Options

- A (171.24, 201.26) B (173.29, 199.21) C (175.19, 197.31)
D (170.77, 201.73) E (173.61, 198.89) F (174.80, 197.70)
-

Questions 18 to 24 are on Chapter D4.

Questions 18 to 20

The salaries of employees (in £) in a certain small company are as follows.

9829	12 554	30 802	23 664	17 582
41 541	29 263	27 486	45 859	14 291

18 Choose the option that is the median salary in £ in the company.

Options for Question 18

A 9829	B 23 664	C 25 287
D 25 575	E 27 486	F 45 859

19 Choose the TWO options that give the lower and upper quartiles for the salaries in the company.

Options for Question 19

A 9829	B 14 291	C 17 582
D 29 263	E 30 802	F 45 859

20 Choose the option that gives the range of the salaries in the company.

Options for Question 20

A 9829	B 18 015	C 25 287
D 25 575	E 36 030	F 45 859

Questions 21 and 22

A small supermarket wishes to test the effectiveness of two types of coupon for a day. The amount (in £) spent by each customer using a coupon is recorded.

A summary of the results is given in the following table.

Coupon	Amount (in £) spent during the day		
	Sample size	Sample mean	Sample standard deviation
Type I	38	26.27	4.37
Type II	32	24.05	9.4

- 21** Choose the option that is closest to the estimated standard error (ESE) of the difference between the sample means for the first and second types of coupon.

Options for Question 21

- A** 0.11 **B** 0.64 **C** 1.81
D 3.26 **E** 6.02 **F** 10.37

- 22** Choose the option that is closest to the magnitude of the test statistic z that would be used to carry out a two-sample z -test to determine whether there is a difference between the mean amount spent with the two types of coupon.

Options for Question 22

- A** 0 **B** 0.06 **C** 1.11
D 1.23 **E** 2.22 **F** 27.80
-

Questions 23 and 24

A production manager in a certain engineering company uses a regression line to model the relationship between production volume and production cost. The regression line obtained using the recorded data has the equation

$$y = 4.4 + 3.2x,$$

where y is the production cost (in £000s), and x is the units produced (in 000s).

- 23** Choose the option that gives the estimated production cost (in £000s) for producing 5500 units.

Options for Question 23

- A 17.60 B 22.00 C 27.40
D 5500 E 17 604.40 F 17 600 004.40

- 24** Choose the option that gives the estimated increase in the production cost (in £000s) due to a production increase from 4000 to 5000 units.

Options for Question 24

- A 3.2 B 4.4 C 10
D 1000 E 3200 F 4400
-